

another wall of the annulus impermeable (i.e. $A = 0$ if it is inner wall and $B = 0$ if it is outer wall of the annulus). It means there exists a suction cross flow either through the inner wall or injection through the outer wall of the annulus.

The convergence rate of the quasilinearization method is fairly fast, only a few iterations are needed to obtain a five digit accuracy. The method gives quite accurate results for moderately large values of Reynolds number up to the desired accuracy and hence may be very useful in solving two-point boundary value problems in the case of non-Newtonian fluids.

The variation of the temperature profile at $A = 1.0$, $B = 0$, $\tau_2 = 0.02$, $\zeta = 0.4$, $P = 0.4$, $E = 1$ for $R = 0.1, 1, 10, 15, 20$ is represented in Fig. 1. It is evident that for small Reynolds number, temperature increases linearly with ξ throughout the annulus. In the case of moderately large Reynolds number, the temperature increases very rapidly at first and then starts decreasing rapidly thereafter. It is also seen that an increase in R increases the temperature very rapidly near the inner wall of the annulus while increases very slowly near the outer wall of the annulus. Hence there is a heating near the inner wall of the annulus which increases with an increase in R .

Figure 2 shows the variation of the temperature profile at $R = 10$, $\tau_2 = 0.02$, $\zeta = 0.4$, $P = 0.4$, $E = 1$ for $A = 0, 0.2, 1.0$ when $B = 0$ and for $B = 0, 0.2, 1.0$ when $A = 0$. In case of $A = B = 0$ the temperature increases with an increase in ξ throughout the annulus. Though in both the cases $B = 0$ and $A = 0, 0.2, 1.0$; $A = 0$ and $B = 0, 0.2, 1.0$ the temperature increases rapidly near the inner wall of the annulus but the rate of increment with A is much faster than that with B . In other words there is more heating near the inner wall in case of suction on the inner wall than that for the injection at the outer wall of the annulus.

Variation of the temperature profile at $R = 10$, $A = 1$, $B = 0$, $\zeta = 0.2$, $P = 0.4$, $E = 1$ for $\tau_2 = 0, 0.001, 0.005, 0.02$ is shown in Fig. 3. It is clear from the figure that the temperature increases rapidly with an increase in τ_2 near the

inner wall of the annulus and decreases for $\tau_2 = 0.001$ and 0.005 to that for $\tau_2 = 0$ thereafter. It is also evident that there is more heating near the inner wall in case of Reiner-Rivlin fluid than that for a Newtonian fluid.

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The general characteristics of boiling heat transfer from a surface embedded in a porous medium

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REVIEW OF THE LITERATURE

IN 1985, Afgan *et al.* [1] conducted experiments for nucleate boiling of water, ethyl alcohol and Freon-113 using horizontal tubes covered with porous layers consisting of scintered dendrite shaped and spherical 63–100 μm particles. In one set of their experiments, the heat flux q vs the degree of superheat or 'excess temperature', ΔT_x , was obtained for a 16 mm diameter electrically heated tube covered by a 2.2 mm thick porous layer saturated with water. In this experiment, they observed that the boiling process can be divided into three regimes. In the first regime, called mode I, which corresponds to relatively small ΔT_x , normal bubble boiling takes place. The second regime, called the transition regime,

occurs at higher ΔT_x . The investigators observed that, in the transition regime, the temperature difference increased by a large increment for a constant value of heat flux after a sufficiently long time interval. They attributed this phenomenon to the slow growth of a vapor film through the porous layer.

After the transition regime, a third regime, called mode II, starts. In this regime ΔT_x increases with increasing heat flux, there is a vapor film inside the porous layer and nucleate boiling occurs at the outer surface of the porous layer. The transition from mode I to mode II can follow along different paths if the time necessary to achieve steady state is not allowed. Mode II persists until a boiling crisis or 'burn out' point is reached.

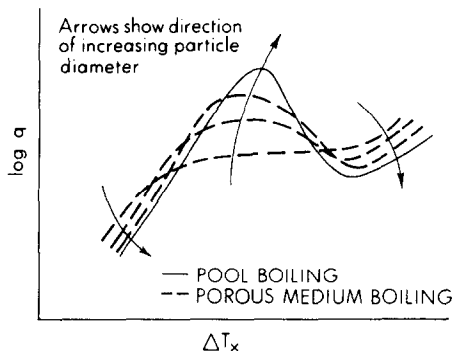


FIG. 1. Sketch showing effect of porous medium particle diameter on porous medium boiling (based on data in Figs. 3 and 4 in ref. [2]).

Fukusako *et al.* [2, 3] have published the results of an extensive experimental study of boiling heat transfer in liquid-saturated porous beds. Figure 1 is a schematic diagram which shows certain general trends of the experimental results presented by Fukusako *et al.* [2]. They found that for relatively small particle (sphere) size the boiling curve does not pass through a maximum, as is the case for pool boiling. In ref. [2] this behavior is explained as follows:

"As the ΔT_x level is increased, it is expected that the number of spots on which bubbles start become larger, and that the heating surface is densely populated with bubbles. Thus, for small bead [sphere] diameters, the supply of liquid to the heating surface through small passages among the beads appears to become choked. Furthermore, to remove the vapor bubble from the heating surface, vapor pressure must overcome the frictional resistance of flow through the small passages of the porous structure. Therefore, for beds composed of small beads the difficulty in the removal of vapor bubbles appears to be caused by a sequence of the facts mentioned above."

Fukusako *et al.* found that the porous medium boiling curve approaches the shape of the pool boiling curve more and more closely for increasing particle sizes as indicated in Fig. 1. Tsung *et al.* [4] observed this same general behavior for boiling heat transfer from a sphere embedded in a porous medium (see Fig. 2 in ref. [4]).

EXPERIMENTAL APPARATUS AND PROCEDURE

Apparatus

In order to obtain meaningful data at atmospheric pressure with water as the saturating liquid, it was necessary to construct an apparatus that could operate at fairly high temperatures (up to 800°C). To this end, a test specimen made of Inconel 600† hypodermic tubing (o.d. = 1.93 mm, i.d. = 1.32 mm) was used. The tubing was suspended horizontally and was heated by passing electric current through it. In conjunction with this test specimen, a porous medium with a matrix composed of spheres made of borosilicate glass which has a melting point of 800°C was employed.

†Inconel 600 has a melting temperature in excess of 800°C.

‡ Calculations show that the difference between the inside and outside surface temperatures of the test specimen were negligible.

Borosilicate glass spheres are relatively costly (more than ten times as costly as the more common soda-lime glass spheres), and hence the present apparatus was designed in such a way as to employ a relatively small quantity of borosilicate glass spheres and yet, at the same time, provide a porous medium sufficiently large as to approximate an infinite medium relative to the size of the test specimen. This was accomplished by building a porous medium reservoir wherein only those glass spheres in close proximity to the test specimen are of the borosilicate kind, while those further away, where temperatures are moderate, are composed of soda-lime glass spheres having the same diameter (3 mm).

A sketch of the apparatus is shown in Fig. 2. It consists basically of a vertical circular water tank, closed at the bottom and open to the atmosphere at the top, which contains a cylinder composed of three segments placed concentrically within the tank. An electrical resistance heater, which was wrapped around the tank, maintained water contained therein at saturation temperature. The central part of the segmented cylinder contained the test specimen and borosilicate glass spheres, while the lower and upper parts contained soda-lime spheres. The borosilicate glass spheres were separated from the soda-lime glass spheres by two thin stainless steel screens, and hence the assembly can be considered to comprise a single relatively large ('infinite') continuous porous reservoir. An insulated chromel-alumel thermocouple located on the centerline of the tubular test specimen was used to measure the average surface temperature of the test specimen.‡ A copper-constantan thermocouple located 10 cm below the test specimen measured the bulk temperature.

Supplementary equipment includes a 110A d.c. power supply for heating the test specimen, a copper coil condenser for condensing steam generated by the boiling process and an interconnected stainless steel auxiliary tank equipped with a sight glass to which make-up water was added at a rate sufficient to maintain a constant water level inside the main tank wherein boiling occurred. The current through and voltage across the test specimen were measured and these data permitted the determination of q .

Procedure

Pool boiling experiments were performed in order to obtain data with which subsequent porous medium boiling results could be compared. The porous medium boiling experiments were performed in two stages. In the first stage experiments with relatively small excess temperatures were performed in order to obtain data comparable to pool boiling in the natural convection and nucleate boiling regimes, and also to determine whether a 'critical point' exists for porous medium boiling. To accomplish the latter objective, the power input to the test cylinder was progressively increased in small increments until, as it was found, a sudden large increase in the wall temperature of the test section was observed. This implied that a critical heat flux had, indeed, been reached. Having verified the existence of the critical point, the second stage ensued, whose purpose was to explore the 'post-critical' region, subsequently identified as the 'transition' region.

Several sequences of tests were made during the first and second experimental stages in order to determine the repeatability of the data. Within each sequence, q was progressively decreased after maximum q had been reached and data were recorded in order to determine the pattern of the reverse heating process. The experiment was terminated after the temperature of the test surface had exceeded the melting temperature of the borosilicate glass spheres.

EXPERIMENTAL RESULTS AND DISCUSSION

Figure 3 is a graph of the experimental data. The scattering exhibited by the porous medium boiling data is attributed

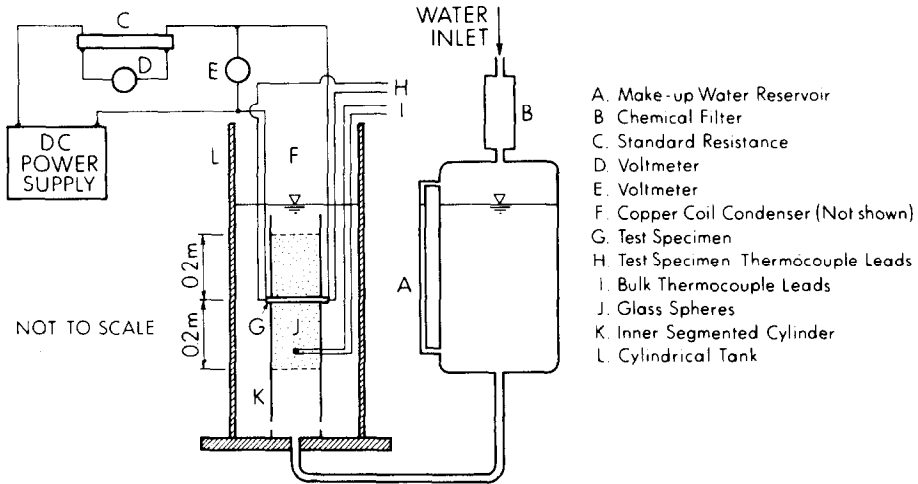


FIG. 2. Diagram of the apparatus.

primarily to variations in the test surface conditions with time due to aging, corrosion and deposition of impurities contained in the water. In spite of the scattering in the data of porous medium boiling in Fig. 3, it is possible to discern certain definite trends. These trends are indicated clearly by the fared curves based upon the data in Fig. 3 which are shown in Fig. 4.

Figure 4 leads to the conclusion that the natural convection regime for the present pool boiling experiments corre-

sponds to the region for which $\Delta T_x \leq 9^\circ\text{C}$. Since the heat flux data obtained for porous medium experiments lie below those obtained for pool boiling experiments for $\Delta T_x \leq 8^\circ\text{C}$, it is concluded that the glass spheres around the test cylinder in the present study did not initiate nucleation when $\Delta T_x \leq 8^\circ\text{C}$ but did affect (diminish) the rate of heat transfer by natural convection by impeding the convection current near the surface of the test specimen.

Figure 4 shows a 'crossover' of the data for porous medium

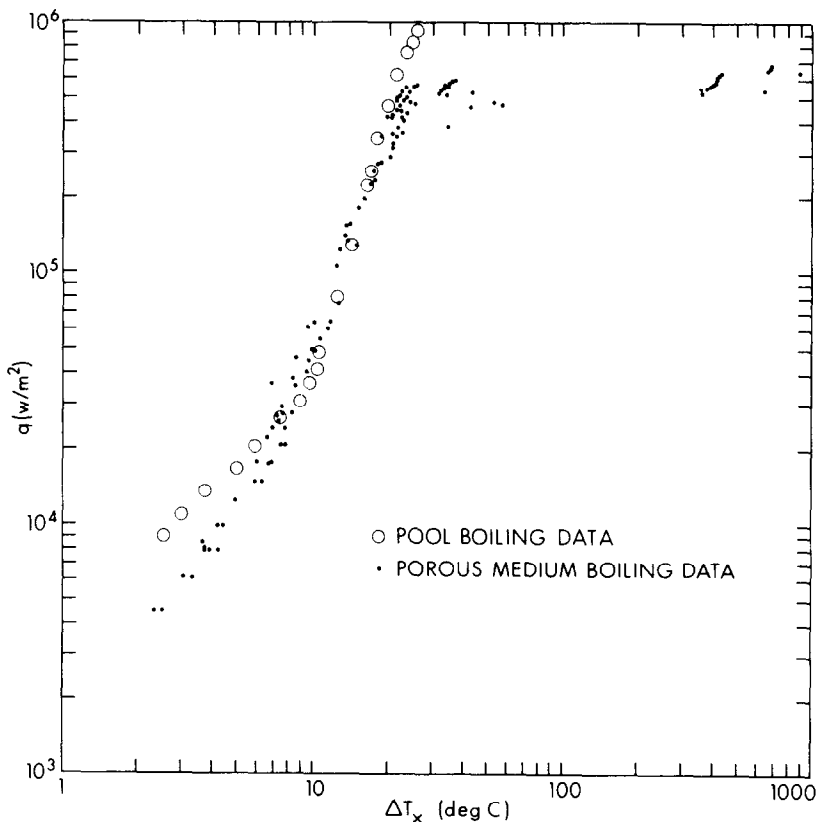


FIG. 3. Plot of heat flux vs ΔT_x for porous medium boiling.

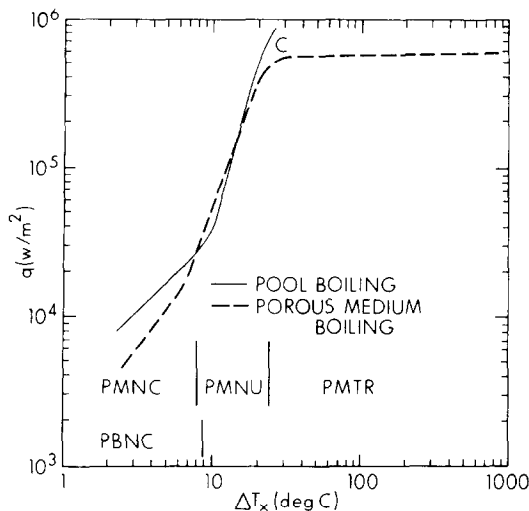


FIG. 4. Trends of the present data (based upon Fig. 3).

boiling with those for pool boiling at $\Delta T_x = 8^\circ\text{C}$. From this it is inferred that the insertion of the glass spheres in the present study caused nucleate boiling to occur at a slightly lower value of ΔT_x (order of 1°C) than occurred for pool boiling. A second 'crossover', which is discussed below, takes place at approximately $\Delta T_x = 17^\circ\text{C}$.

In Figs. 3 and 4 it is observed that for $9 < T_x \leq 17^\circ\text{C}$ the data obtained for porous medium boiling lie close to those for pool boiling. Thus, in this region, the growth and collapse of vapor bubbles on the heated surface is not significantly affected by the insertion of glass spheres into the region surrounding the test specimen. This observation, which at first may be surprising, is a result of the fact that the glass spheres in this experiment are 1.7 times as large in diameter as the test specimen. Therefore, the dimensions of the fluid filled volumes in the vicinity of the test specimen are relatively large, even with the glass spheres in place, and since the vapor bubbles in the early portion of the nucleate boiling regime ($9 < \Delta T_x \leq 17^\circ\text{C}$) fill a small portion of the fluid volume, the net effect is that the heated surface is nearly unaffected by the presence of the glass spheres; in other words, for $9 < \Delta T_x \leq 17^\circ\text{C}$, the nucleate boiling processes are basically the same whether or not glass spheres are present for the geometry and materials studied here.

Figure 4 shows that when $17 < \Delta T_x \leq 25^\circ\text{C}$, the heat flux obtained with glass spheres lies below the value obtained in pool boiling. It is postulated that, at this point in the nucleate boiling regime, the vapor bubbles are comparable in size to the volumes between the glass spheres and test specimen, and hence the spheres impede the detachment and motion of these bubbles due to surface tension effects. Again, the result is a reduced rate of heat transfer compared to pool boiling.

Point C on Fig. 4, corresponding to $\Delta T_x = 25^\circ\text{C}$, is a 'critical point' in the sense that beyond this point ΔT_x increases rapidly if q is increased slightly. Further, it is apparent that when ΔT_x increased from 25 to 900°C as indicated in Fig. 3 a transition to film boiling must have occurred. It follows that the increase in ΔT_x during this transition must be caused by an increase in the fraction of the total heated surface area that is covered by vapor. The transition to film boiling for the porous medium corresponds to a curve that has a very shallow slope on a plot of q vs ΔT_x over an extended range of ΔT_x (see Fig. 4). In this transition regime the value of ΔT_x was observed to fluctuate randomly by as much as 100°C about a mean for an imposed

constant value of q . This fluctuation was attributed to the random motions of vapor bubbles in the interstitial spaces between the test specimen and the randomly packed glass spheres. Test results were identical if q was increased or decreased sufficiently slowly. After the last datum plotted in Fig. 3 ($\Delta T_x = 900^\circ\text{C}$) was obtained, test results differed from those obtained before and the experiment was terminated. Subsequent inspection revealed that small spots of borosilicate glass had fused to the surface of the test specimen.

COMPARISON OF PRESENT RESULTS WITH PREVIOUS STUDIES

The discussion which follows involves comparisons between heat transfer results obtained in the presence of porous matrices with comparable results obtained for pool boiling. To facilitate the discussion, the following abbreviations will be used to designate the various boiling regimes:

- PBNC — pool boiling natural convection regime
- PBNU — pool boiling nucleate regime
- PBTR — pool boiling transition regime
- PBSF — pool boiling stable film regime
- PMNC — porous medium boiling natural convection regime
- PMNU — porous medium boiling nucleate regime
- PMTR — porous medium boiling transition regime
- PMSF — porous medium boiling stable film regime.

The range of the excess temperature for a particular regime will be denoted by prefixing the designations defined above by ΔT_x ; thus, for example, ΔT_x (PBNC) represents 'the range of ΔT_x for the pool boiling natural convection regime'. Several of these ranges are indicated in Fig. 4.

A comparison of Fig. 1 with Fig. 4 reveals that in the present study, the presence of the porous medium reduced the heat transfer rate in most of PBNC (up to $\Delta T_x = 8^\circ\text{C}$), whereas it increased the heat transfer rate throughout PBNC in the prior study by Afgan *et al.* Obviously, the porous layer surrounding Afgan *et al.*'s test cylinder induces nucleation at substantially lower values of ΔT_x than occurred in the absence of the porous medium, contrary to what was encountered in the present study for the same basic cylindrical geometry. Further, Fig. 1 shows that the smaller the interstitial volumes, the greater the increase in nucleation activity and heat transfer rate compared with PBNC. This observation is consistent with the known fact that small crevices in a surface become nucleation sites for boiling. These comparisons of experimental results substantiate the intuitive hypothesis that the characteristic dimensions of the interstitial volume between particles of a porous medium and between those particles and an embedded heated surface have a major influence on the boiling process at and near the surface.

The behavior depicted in Fig. 4 in PMNU is qualitatively similar to that depicted in Fig. 1 corresponding to the curve for the smallest particle diameter. In the lower portion of ΔT_x (PMNU), the heat transfer rate is higher than for pool boiling; and, in the higher portion of ΔT_x (PMNU), the heat transfer rate is lower than for pool boiling, thereby creating a 'crossover point'. It is postulated that, for values of ΔT_x below the crossover point, the porous medium increases the heat transfer rate by increasing the pumping effect associated with the generation and detachment of bubbles at the surface, owing to a decrease in volume available for fluid flow in the case of porous medium boiling compared with the case for pool boiling. However, the crossover represents a 'point of diminishing returns' with respect to heat transfer inasmuch as for ΔT_x beyond the crossover point, the size and number of bubbles are so large and bubble detachment is so impeded by the additional surface tension forces created by the presence of the particles of the porous medium, as compared

with surface tension forces present in pool boiling, that the net effect is to 'insulate' the surface with vapor relative to pool boiling and thereby reduce the heat transfer rate compared to pool boiling.

The insulation effect described above continues to operate in PMTR, except that now there are portions of the heat transfer surface which are continuously covered by vapor, and these portions represent an increasing fraction of the total heat transfer surface as ΔT_x increases. Eventually, the entire heat transfer surface becomes covered with vapor and this condition is called stable film boiling. The heat transfer rate, which is nearly constant in PMTR for sufficiently small particles begins to rise again in PMSF with increasing ΔT_x (see Fig. 1 for smallest size particles) for obvious reasons. As stated earlier, the behavior in PMNU, PMTR and PMSF approaches that in PBNU, PBTR and PBSF for increasing particle sizes (again, see Fig. 1).

Fukusako *et al.* [2] found that their heat transfer results were unaffected by changing the depth of the porous bed from 30 to 300 mm. They concluded: "This suggests that the essential mechanism of boiling heat transfer is determined in the comparatively thin layer of beads [glass spheres] near the heating surface." In this connection it is interesting to observe that Afgan *et al.*, who employed a much thinner porous layer than did Fukusako *et al.* (and, of course, much thinner than in the present study) encountered a phenomenon not observed by Fukusako *et al.* or by the present investigators. This phenomenon occurs when the porous layer is completely filled with a stable film and now the test cylinder together with its porous cover comprises a single entity which eventually reaches the 'burn out' point.

It is important to note that, although certain trends of the data in the present study are similar to those in ref. [2], the magnitudes of the heat transfer rates in the two studies are quite different for the same values of ΔT_x . Thus, for $\Delta T_x = 25^\circ\text{C}$ the value of $q \cong 6 \times 10^5 \text{ W m}^{-2}$ in the present study, but $q \cong 4 \times 10^4 \text{ W m}^{-2}$ for $\Delta T_x = 25^\circ\text{C}$ for a comparable porous medium in ref. [2]. Therefore, the correlation equations in ref. [2] are not applicable to the geometry employed in the present study.

CONCLUSIONS

The experimental data and discussions thereof presented above indicate that boiling heat transfer from a surface embedded in a porous medium is a very complex process that depends upon many variables. An analysis of this process would require consideration of as many as eight heat transfer paths, enumerated as follows.

- (1) Conduction heat transfer within the heated specimen.
- (2)–(4) Heat transfer from the surface of the heated specimen to porous medium particles, to saturating liquid and to vapor.
- (5) Heat transfer from attached (superheated) vapor films to porous medium particles.
- (6), (7) Heat transfer from porous medium particles to the saturating liquid and to (or from) vapor bubbles.
- (8) Heat transfer from vapor films and bubbles to the saturating liquid.

If, in addition to the complexity of the heat transfer paths, one considers the additional complexity of the geometry of the interstitial volumes of the porous medium; and further, if one considers the fact that the bubble and vapor film dynamics depend not only upon the thermophysical properties of the vapor and liquid but also upon the shapes of the interstitial volumes plus several different values of surface tension, it appears that a theoretical analysis of this problem is well nigh unattainable.

In ref. [2] Fukusako *et al.* have done an admirable piece of empirical work in correlating their heat transfer data in the transition and film boiling regimes, but their correlations are applicable to a very specific geometry and do not apply generally. The fundamental reason why their correlation equation cannot be expected to apply more generally is that their equations do not contain certain parameters upon which the fluid mechanics of vapor generation and motion in a porous medium depend, such as, for example, the surface tension along the lines of contact between a vapor bubble and particles of the porous medium. Therefore, it appears that experimentation with an appropriate model is necessary if design data is needed for a particular application.

Stated below are two general characteristics of boiling heat transfer from a surface embedded in a porous medium that are considered to be of prime importance.

(1) The presence of a porous matrix causes nucleation to occur on a heated surface at lower values of ΔT_x than in pool boiling—the smaller the matrix's interstitial volumes (and hence the greater the density of contact points between the particles of the matrix and the heat transfer surface), the earlier the onset of nucleation. Earlier nucleation causes a large increase in the heat transfer rate, which would otherwise be controlled by natural convection.

(2) A porous matrix composed of sufficiently small particles will cause ΔT_x to increase monotonically with q as shown in Figs. 1 and 4. As the size of the particles of the porous matrix are increased, the boiling curve approaches that for pool boiling as shown in Fig. 1. This conclusion is probably the most important single piece of information gained thus far from the study of boiling in porous media, because it means that the sudden jump in temperature to 'burn out' that occurs in pool boiling can be avoided in porous media.

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